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# THE AMERICAN MATHEMATICAL MONTHLY

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## THE TEACHING OF MATHEMATICS.

By H. E. SLAUGHT.

Owing to a wide divergence of opinion among college and university men as to the proper interpretation of the term *pedagogy*, and especially in view of the varying notions of that term in the minds of mathematicians, it seems desirable to formulate as definitely as possible what is conceived to be the relation of the MONTHLY to the question of *teaching* in the field of collegiate mathematics.

A certain kind of formal didactics, which at one time prevailed widely in schools for the training of elementary teachers, and which also characterized the earlier attempts at pedagogical training for secondary teachers, has served to alarm many college and university men, whenever the question of college teaching is mentioned, lest the same stilted and formal methods be carried over into the collegiate field. These men have seen some horrible example of a college teacher using what they describe with fine scorn as "high school methods," and thereby sacrificing the higher spirit of the subject to the lower criterion of *so-called pedagogical form*. They say, and with good reason let it be granted, that they would far rather have a young college instructor with good training, fine spirit, and high ideals, who had no teaching experience and no pedagogical training, rather than one whose initiative and individuality is so hedged about with formal rules on teaching and methods of presentation that he becomes a mere machine. And having thus spoken, these people feel that they have said the last word on the teaching of mathematics so far, at least, as the collegiate field is concerned. They think that the teacher is born, not made, that if the young fellow has it in him he will become a good teacher after a certain period of floundering, and if it is not in him, no amount of pedagogical training can make him a good teacher.

There is so large an element of truth in the foregoing argument that many are fully convinced by it. It is true that a bright student from the graduate school may go directly to the teaching of college freshmen and may eventually become proficient at the work, just as it is also true that a student may go directly from the medical school to the independent practice of medicine without the professional training of the clinic and the hospital internship.

It is abundantly evident to the impartial observer that the teaching of college freshmen and sophomores is not so widely different from the teaching of high school juniors and seniors; and that, on the whole, there is as large a proportion of poor teaching done in the colleges as in the high schools; with the certainty that this proportion will rapidly increase unless remedial measures for the colleges are soon undertaken comparable with those now in operation for the high schools. The logical step forward is a graduate school of education and graduate departments of education which shall give serious and scientific attention to the betterment of college teaching. Meanwhile, so far as mathematics is concerned, the MONTHLY advocates certain general measures which it conceives to be of fundamental and far-reaching importance:

1. **The Stimulation of Individual Initiative.** It is, of course, recognized that in the last analysis good teaching depends upon the initiative of the individual, and to stimulate such initiative is a prime object of the MONTHLY. It is believed that the best way to accomplish this is not by direct reference to the daily routine, nor by dwelling upon pedagogical tricks, but by bringing to the teacher's attention regularly and systematically, in attractive form, the liveliest discussions possible concerning all phases of work in which he is engaged and in which he should, by virtue of his position, be interested. To this end the MONTHLY has brought to its readers, since its reorganization in 1913, two exceedingly stimulating series of articles by Professor Cajori, one on the History of the logarithmic concept and the other on the History of Zeno's arguments, being phases in the development of the theory of limits, besides half a dozen other equally valuable articles on historical subjects. It has provided no less than thirty articles relating to the teaching of mathematics, the nature of which may be inferred from a few titles taken at random: "Mathematical troubles of the freshman," "Incentives to mathematical activity," "Synthetic projective geometry as an undergraduate study," "On the cultural value of mathematics," "On courses in the history of mathematics," "Determinant formula for the coplanarity of four points," "Some things we wish to know," "What can the colleges do toward improving the teaching of mathematics in the secondary schools?", "Conference periods for college students," etc.

Other lines of personal stimulus have been offered through numerous contributions of general mathematical information, such as the various meetings of mathematicians in this country and in Europe, for example, the "Napier tercentenary celebration," and the "Paris report on calculus in the secondary schools"; through an abundance of shorter notes on matters of mathematical interest, such as comments on various publications, and excerpts from foreign journals in this field, for example, the "Life of Pythagoras" condensed from an exhaustive article by Professor W. W. R. Ball in the *Mathematical Gazette*; and by critical and helpful reviews of new mathematical books in the collegiate and advanced secondary fields.

Finally, in addition to these many forms of stimulus of a more or less indirect nature, the MONTHLY has sought to bring to the individual teacher a direct and

cogent appeal to personal initiative by providing a long list of mathematical papers that involve a minimum of technical treatment and presuppose, for the most part, only the topics usually given in undergraduate courses of study; and a shorter list of papers, involving more technical treatment, for the benefit of those who are ambitious to press on into somewhat higher ranges of mathematical reading. The object of the MONTHLY in selecting these papers is not simply to provide entertainment of a high character, nor even to be content with arousing *passive* interest in this intermediate field, but to *stimulate activity* on the part of college teachers—activity that may be reflected in the class room in the form of renewed enthusiasm, of incisive and critical presentation, of lively and keen interest that is always contagious among the students; and activity that may lead to *production*, to the contribution of similar papers for the benefit of others. If this supreme ambition on the part of the MONTHLY, which is meeting with most encouraging fulfillment in a narrow but gradually widening circle of influence, shall become in anywise general, then we believe that a fundamental and permanent contribution will have been made to the betterment of teaching in the colleges. To this end, every encouragement has been offered to all who will make even a small beginning. The department of Questions and Discussions provides an opportunity for minor contributions of varied character, and the department of Problems makes an appeal to large numbers. The really careful reviewing of a book is an activity that may easily lead to fruitful results. The writing of an acceptable article, historical, critical or technical, may be the beginning of a new epoch for the author; certainly persistence in such activity is capable of transforming a career, as attested by many notable examples.

**2. The Stimulation of Group Organization.** However much depends upon individual initiative and activity, it is certain that group activity is a kind of stimulus that seems to be necessary for the accomplishment of large things. There must be intercommunication and action based upon interchange of ideas. For this reason the MONTHLY has embraced every opportunity to encourage those organizations that have for their object the kind of development above described. For example, the reading club of the California teachers and the new organization among those secondary teachers of New Jersey who wish to discuss mathematical subjects at their meetings rather than mere pedagogical questions. But the aim of the MONTHLY is to see the college teachers of mathematics organized in every state, or even in some smaller groups, for the purpose of mutual assistance along all lines of interest in the collegiate field. A number of such groups are already under consideration, and one is being formally organized in the state of Kansas as this issue goes to press. The discussion at this meeting is on question 27 in the September MONTHLY, namely,

“A certain college wishes to offer twelve hours of mathematics beyond the usual courses in analytic geometry and calculus. Considering only the needs of students intending to specialize in pure mathematics, what courses should make up the twelve hours offered?”

The MONTHLY congratulates these Kansas teachers and commends their example to all other such groups of college teachers throughout the country. With the

attention of college men fixed upon the varied and special problems of college work in mathematics, and with many group organizations for interchange of ideas, a new era will be inaugurated in this field.

3. **The Stimulation of National Organization.** The most natural climax of wide group organization is a national organization, such as was referred to in the October issue, and conversely, such a national organization will react on the formation of smaller groups and both will provide a far-reaching stimulus to individual activity. The MONTHLY would have welcomed the incorporation of such an organization within the American Mathematical Society, but since this is not to be, we look forward with high hopes and great enthusiasm to the organization of a new national society, and with more than four hundred charter members, we see no reason why commendable things should not be accomplished through this movement for the cause of mathematics in America.

## HISTORY OF ZENO'S ARGUMENTS ON MOTION:

### PHASES IN THE DEVELOPMENT OF THE THEORY OF LIMITS.

By FLORIAN CAJORI Colorado College.

#### X.

##### E. POST-CANTORION DISSENSIONS (Concluded).

With the advent of the new century, discussion on Zeno began to quiet down in France. We note only two articles. In 1907 O. Hamelin wrote on the "Arrow," but the interest of his article centers in what constitutes the most probable renderings of the Aristotelian text.<sup>1</sup> In 1909 a novel attempt to solve Zeno's puzzles was made by Dunan<sup>2</sup> in an article in which he retracts what he said on this subject in a pamphlet of 1884.<sup>3</sup>

He believes that the difficulties vanish, on the recognition that motion takes place through a space, one and indivisible, without succession and parts. He admits that such a proposition raises considerable difficulty, which cannot be removed except by long and elaborate metaphysics, of which he gives in his article only a bare sketch.

No less radical is the position of Henri Bergson. He holds that philosophy must get back to reality itself. Reality is supplied by intuition. Pure intuition, external or internal, is that of undivided continuity. Every movement, in as much as it is a passage from rest to rest, is in fact absolutely indivisible. Sight perceives the movement in the form of a line which is traversed, and this line, like all space, may be indefinitely divided. We must not confound the data of

<sup>1</sup> *L'année philosophique* de F. Pillon, Paris, 1907, pp. 39-44.

<sup>2</sup> "Zénon d'Élée et le Nativisme" in *Annales de Philosophie Chrétienne*, 1909.

<sup>3</sup> *Les arguments de Zénon d'Élée contre le mouvement*, Nantes, 1884.